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Geochron: a framework to estimate fracturation of deformed sedimentary layers

J.-J. Royer [1], J.-L. Mallet[1], R. Cognot[1] and R. Moyen[1]
1 Gocad Research Group, CRPG, Nancy School of Geology, France
Corresponding author: royer@gocad.org

ABSTRACT: A full-3D balanced restoration technique is used to estimate the fracturation of a deformed sedimentary layer using a Geochron parametric representation of the ante-deformed geological structure. The displacements and the strains which have affected the geological formation are computed assuming small deformations and using the elastic solid theory of continuous media. In simple cases such as thin plates, this new approach is in agreement with results predicted by the theory. Strain tensor invariants (dilatation coefficient, principal strains, etc.) coupled with mechanical properties of rock types are used to characterize the distribution of fracture orientations. It also gives useful strain parameters that can be related to the observed faults. This general theoretical framework provides solutions to complex problems such as the determination of strains resulting from tectonic events and for predicting faulted zones in reservoirs. The method is applied on the Split Mountain Anticline Case Study (Utah, USA), a natural outcropped clastic reservoir. Observed fractures on the field are in good agreement with the ones predicted from the proposed restoration methodology. The final goal of this research is to improve oil and gas recovery in fractured reservoirs by a better estimation of the permeability tensor.

KEYWORDS: trend, anisotropy, membership function, categorical variable simulation.

1. Introduction

3D unfolding techniques are common practices to understand history of fracturing in complex faulted reservoirs, especially for oil and gas exploration. Using the solid theory of continuous media, we propose to derive several structural attributes (dilatation coefficient, principal strains, etc.) that may be used to define fractured zones using failure criteria. This approach quantifies deformations and gives good results both in theoretical models and real case-studies. It also provides an estimate of strains and stresses during deformation. These parameters can be used to predict faulted zones in reservoirs and better estimate permeability.

2. Theoretical background

2.1. Deformation analysis in a nutshell

Elastic solid theory relates the variation of the metric tensor to the strain tensor of continuous media $\boldsymbol{\varepsilon}(\boldsymbol{x}) = ½\,[\boldsymbol{g}(\boldsymbol{x}) - \boldsymbol{g}_0(\boldsymbol{x})]$ where $\boldsymbol{\varepsilon}(\boldsymbol{x})$ is the strain tensor which characterizes the deformation of elastic solid in the neighbourhood of location $\boldsymbol{x}$ between depositional time t_0 and present time t; $\boldsymbol{g}(\boldsymbol{x})$ and $\boldsymbol{g}_0(\boldsymbol{x})$ are the metric tensors of the present state solid and of the restored one, respectively (see Sedov, 1973; Mallet, 2002; Royer et al., 2003). The metric tensor measures distances between points as the solid is deformed through time. The metric tensor $\boldsymbol{g}_0(\boldsymbol{x})$ is assumed to be diagonal, while $\boldsymbol{g}(\boldsymbol{x})$ is estimated either from displacement restoration vectors (Muron, 2005) or, equivalently, from a curvilinear coordinate system attached to the deformed object (called *Geochron Model* see Royer et al., 2003; Mallet, 2004). The components of the strain tensor may differ depending on the coordinate systems. Several structural parameters can be derived from the metric tensors such as:

(i) The *cubic dilatation coefficient* $\theta(\boldsymbol{x})$, an invariant geometric characteristic that measures the variation of volume of an infinitesimal element of volume dV during deformation:

$$\theta(\boldsymbol{x}) = (dV(\boldsymbol{x}) - dV_0(\boldsymbol{x})) \,/\, dV_0(\boldsymbol{x}) = [det\,(\boldsymbol{G}(\boldsymbol{x}))/det\,(\boldsymbol{G}_0(\boldsymbol{x}))]^{1/2} - 1 \qquad (1)$$

Where *det* is the determinant. In the case of small deformations, it can be shown (Sédov, 1974) that: $\theta(\boldsymbol{x}) = tr\,(\boldsymbol{g}^{-1}(\boldsymbol{x})\,\boldsymbol{\varepsilon}(\boldsymbol{x}))$ (where *tr* is the trace);

(ii) The *principal strains* and their directions (called *principal strain directions*) are given, respectively, by the *eigen values* λ_i (i=1, 2, 3) and the *eigen vectors* $\boldsymbol{v}_i$ of the Eulerian strain tensor $\boldsymbol{S}(\boldsymbol{x}) = \boldsymbol{g}^{-1}(\boldsymbol{x})\,\boldsymbol{\varepsilon}(\boldsymbol{x})$. The principal strain directions are orthogonal and independent on the coordinate system.

$$\boldsymbol{S}(\boldsymbol{x})\,\boldsymbol{v}_i(\boldsymbol{x}) = \lambda_i\,\boldsymbol{v}_i(\boldsymbol{x}) \qquad \forall\ i = 1,2,3 \qquad (2)$$

(iii) The *relative elongation coefficients* $l_i(\boldsymbol{x})$ along the principal directions are also invariant and are deduced from the eigen values $\lambda_i(\boldsymbol{x})$ of $\boldsymbol{S}(\boldsymbol{x})$ as follows:

$$l_i(\boldsymbol{x}) = [1 - 2\,\lambda_i(\boldsymbol{x})]^{-1/2} - 1 \qquad (3)$$

In the case of small deformations, it can be simplified into: $l_i(\boldsymbol{x}) = \lambda_i(\boldsymbol{x})$.

The above parameters fully characterize the strain at each location $\boldsymbol{x}$. They are independent of the coordinate system (also called *invariant*). In the following, we will refer to them as *structural attributes*. One of the important points is how to restore a structure in a consistent manner in order to estimate the deformation tensor from initial to final states. This will be done using two algorithms: the first one (equivalent to a minimum internal energy criterion) consists in minimizing the strain energy induced by deformations from initial to final states (Muron, 2005); the second one (Moyen, 2005) is based on a local parameterization of volumes, in which isochrones correspond to depositional surfaces (called *Geochron* Model). If the parameterization is consistent with the geology both approaches give similar results.

2.2. Mechanical Model

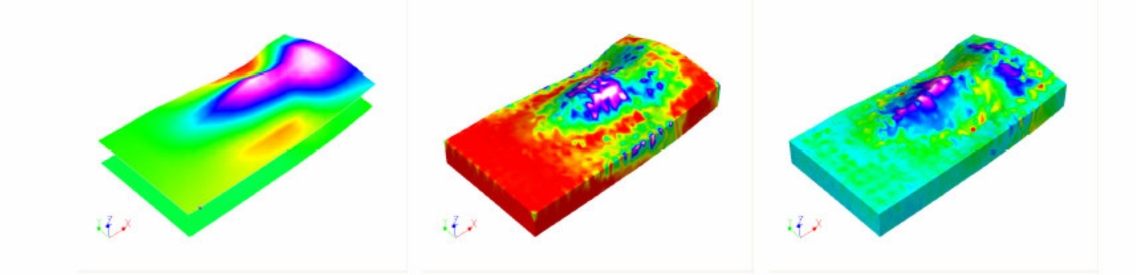
Strains are related to *stresses* by the constitutive law of the material under consideration (geotechnical model). For sake of simplicity, it is assumed here that the rock is linear, isotropic and elastic. It can be described by the Hooke's law which assumes a linear relationship between the stresses σ_{ij} and the strains ε_{ij} according to:

$$\sigma_{ij} = \lambda\,tr\,(\boldsymbol{\varepsilon})\,\delta_{ij} + 2\,\mu\,\varepsilon_{ij} \qquad (4)$$

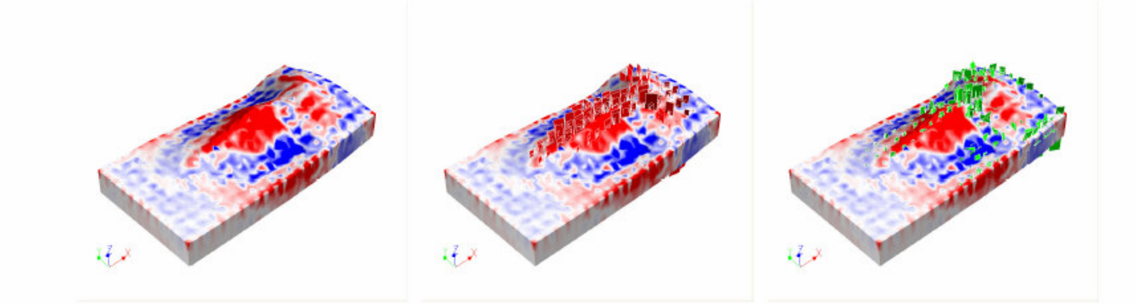
where λ and μ are the Lame coefficients, δ_{ij} is the Kronecker symbol ($\delta_{ii} = 1$; $\delta_{ij} = 0$, $i \neq j$). If the material is isotropic, the principal axes of stress and strain tensors coincide. Several failure criteria can be used to predict fractured zones (Macé, 2006). One of the most in use is the *Mohr-Coulomb criterion* that predicts failure of brittle rocks when the Mohr's Circle at a point exceeds the envelope created by the uni-axial *tensile* and *compression strength* circles. In the case of isotropic homogeneous material, this criterion can be rewritten in terms of strains using Eq. (4) (Royer et al., 2003):

$$(\varepsilon_1 - \varepsilon_3)\,\mu\,/\,cos\,\varphi - (\varepsilon_1 - \varepsilon_3)\,\mu\,tg\,\varphi - tr(\boldsymbol{\varepsilon})\,\lambda\,tg\,\varphi \geq c \qquad (5)$$

where c and φ are respectively the internal cohesion and friction angle of the rock.



(a) Model (b) Principal strains (ε_1) (c) Principal strains (ε_2)



(d) Dilatation (θ) (e) Open faults (f) Slip faults

Fig. 1. Structural attributes computed from 3D unfolding: (a) Initial 3D split Mountain model (vertical scale x 3) Top (colored with elevation z) and Bottom triangulated surfaces constrained by field observations used to build the solid model. Principal strains (ε_1) (b) and (ε_2) (c) computed from a Geochron parameterization of the solid model; (d) Dilatation coefficient (θ); Observed open (e) and slip (f) fractures mapped onto the dilation map. As expected, open fractures occur mostly on the top of the anticline where the rock is in extension (in red), while slip fractures are observed on the flank in zones in compression (in blue).

3. The Split Mountain anticline Case Study, Utah

This methodology was applied on the “Split Mountain” anticline (eastern end of the Uinta Mountains, Utah). The main objectives were to derive structural attributes related to fault occurrences observed in the field for predicting fractures.

3.1. Geological Setting

The “Split Mountain” anticline, in Dinosaur National Monument, Utah, is a manifestation of Late Mesozoic to early Tertiary compression within the Cordilleran region. It formed during the Eocene uplift of the Eastern Uinta Mountains. Interest in fractures at Split Mountain Anticline raises from its similarity to faulted anticline reservoirs in which fractures affect the efficiency of secondary recovery (Silliphant et al., 2002). It therefore constitutes a natural analogue of faulted reservoirs. Field studies were conducted by a consortium involving IFP, Total, ASGA and Earth Decision Sciences. Geological units were identified and described according to their lithology, thickness and style of deformation. Two groups of fractures, strip slip and extension fractures, respectively, were recognized. A 3D model of the Split Mountain formation was built in gOcad using: digital elevation models, outcrop observations, geological maps, and dip measurements from field study, area photos and remote sensing data. The top and bottom surfaces of the main sandstone formations were built after integrating all these data into a comprehensive database.

3.2. Results

Two strain regimes can be observed at Split Mountain from the dilation coefficient map (see Fig. 1): (i) an *extension* type of deformation (in red) on the top of the anticline with an EW orientation in the central part of the structure, and a NW orientation on the east flank of the formation; and (ii) a *compression* type of deformation (in blue) on the longitudinal lateral N and S flanks of the anticline. The main extension strain directions ($\theta > 0$) are consistent with the orientation of the extension fractures reported by field observations, while slip fractures ($\theta < 0$) occurred mainly in compression zones (Fig 1d). The fault intensity seems to be correlated with the dilatation coefficient. These results suggest that structural attributes might be helpful in predicting fractures and occurrence mode. Coupled with a failure criterion, one can predict occurrence map of fractures together with their orientation. This procedure then gives new tools to improve the estimation of the secondary permeability. This is a challenge for the future development of this work.

4. Conclusions

The Geochron Model and the elastic solid theory can be used to derive structural attributes from a parametric representation of a deformed geological structure. Results are in agreement with theoretical cases such as the thin plate (Royer et al., 2003) and provide useful structural attributes (dilatation coefficient, principal direction of strains, etc.) that can be helpful: (i) to predict fracturation and sub-seismic faults; (ii) to derive fault occurrence map when coupled with failure criterion; (iii) to improve estimation of secondary permeability. Future development of such approach should focus on: (i) failure criterion based on rock types; (ii) couple mechanical and geostatistical approaches (structural attributes as secondary predictive variables in co-kriging) for a better estimate of permeability; (iii) continue to calibrate this approach onto real case tests such as reservoirs and natural analogues.

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